# INPUTS & OUTPUTS TO NPEX MAT3D MATERIAL ROUTINES

## Deformation Measure (Input)

The inputs are the components of the Green-Lagrange strain tensor in an orthonormal coordinate system. I.e. the inputs are the components Eij where the Green Lagrange strain tensor is written as

**E** = Eij **G**i **G**j

in which **G**i = **G**i are orthonormal basis vectors (user specified for anisotropic material), implied summation of repeated indexes is used throughout unless otherwise noted, and

Eij = ½ (ui,j + uj,i + uk,i uk,j )

where

ui = ui(X1, X2, X3 )

are the displacements of a point which is at location (X1, X2, X3 ) in the undeformed body, and

(.),i

denotes differentiation with respect to Xi.

In the NPEX routines, these strain components are stored in a 1-dimensional array in the form

(E11, E22, E33, 2E12, 2E13, 2E23 )

## Stress Measure

As output the NPEX mat3d\* routines provide the components of the 2nd Piola-Kirchhoff stress tensor in the orthonormal coordinate system **G**i=**G**i. Specifically, the outputs from the NPEX mat3D routine are the components τij, where the 2nd P.-K. stress tensor is given by

**S** = τij **G**i **G**j

**τ**  = τij **g**i **g**j = (dV/dv) **σ**

In the above, **τ** is the Kirchhoff stress tensor, **σ** is the Cauchy (i.e. true) stress tensor, dV is an infinitesimal volume increment in the undeformed configuration, and dv denotes the volume of the same material in the deformed configuration.

These components are energy-conjugate to the strain components used, and are stored in an array

(τ11, τ22, τ33, τ12, τ13, τ23 )

so that the dot product with of the stress array with the increment in the strain array provides the work done on the material per unit volume in the undeformed configuration.

The relationship between the 2nd P.-K. stress tensor and the Cauchy stress tensor is

**σ** = (dv/dV) **F.S.F**T

where

**F** = (δij + ui,j ) **G**i**G**j

is the deformation gradient tensor and dv/dV is the deformed/undeformed ratio of specific volumes, given by

dv/dV = det **F**

For the decomposition of the deformation gradient tensor of the form **F** = **R**.**U** where **R** represents a (proper orthogonal) rigid body rotation tensor, and **U** a symmetric stretch tensor, one obtains:

**σ** = (dv/dV) **R.U.S. (R.U)** T

**σ** = (dv/dV) **R.U.S. U.R** T