Nelmt22 Notes  
Concrete Coating Element

started 18 Jan 2018 when generalizing to 3D

# Summary

Nelmt22 is a moderte-deflection, 2D or 3D element to model concrete coating over a pipe, with a limited bond strength between the coating and the pipe, according to the formulation by Statoil [1-3], but including geometric nonlinearity with moderate deflections, and 3D capabilities.

The element is compatible with the pipe element nelmt25, and used the same kinematic approximations, and degrees of freedom, except that nelmt22 has an extra degree of freedom for the relative axial slip between of the concrete coating over the pipe.

This element and its initial testing is described in [3]. Generalization to 3D was done starting 14jan2018, supported by Section 2.

# Basics

Nodal Coordinates:  
2D case: X = [X Z]T (length=2)  
3D case: X = [X Y Z]T (length=3)  
X is axial, Y and Z transverse.

Nodal degrees of Freedom:  
2D case: u = [ us w θ s ]T (length=4)  
3D case: u = [ us v w θy θz s ]T (length=6)  
All interpolated linearly over the element to an integration point at the centre of the element, except slip s for which the integration point is at the nodes with a tributary area based on half the element length.

Element degrees of Freedom (nodal dofs at each of 2 element nodes):  
2D case: q = [ us1 w1 θ1 s1 us2 w2 θ2 s2]T (length=8)  
3D case: q = [ us1 v1 w1 θy1 θz1 s1 us2 v2 w2 θy2 θz2 s2 ]T (length=12)  
where  
usi = pipe axial displ at element node i (x-direction)  
si = axial slip of concrete coating over pipe at element node i  
vi, wi = transverse displacements (y & z direction respectively)

Concrete Deformation Vector:  
2D case: qc = [ u1 w1 θ1 u2 w2 θ2 ]T (length=6)  
3D case: qc = [ u1 v1 w1 θy1 θz1 u2 v2 w2 θy2 θz2 ]T (length=10),   
ui = usi + si for i=1,2 (concrete axial displacement)

Strains (in concrete):  
2D case strains ϵ = [ϵ κ γ ]T  
3D case strains ϵ = [ϵ κy κz γy γz ]T  
Shear deformations γ are included, but currently no resistance against γ assumed. I.e. corresponding stress is zero. This is not the most efficient way, but makes it possible to use chunks of nelmt25 code without changes.  
ϵ = axial strain at centroid  
κy = rate of of change of axial strain with transverse coordinate y  
κ = κz = rate of of change of axial strain with transverse coordinate y

# Formulation

## Strain Displacement Relations

The relations between ϵ and qc for nelmt22 are the same as those between ϵ and q for nelmt25.

2D:  
z = Z + w  
ϵ = u’ + ½ (z’2 – Z’2) = u’ + Z’ w’+ ½ w’2   
κ = θ’  
γ = w’ + θ

3D:  
y = Y + v  
z = Z + w  
ϵ = u’ + ½ (y’2 – Y’2)+ ½ (z’2 – Z’2) = u’ + Y’ v’+ ½ v’2+ Z’ w’+ ½ w’2  
κy = θy’ κz = θz’   
γy = v’ + θy γz = w’ + θz

In addition have bond deformations: s = s (i.e. displ and strain are the same).

These strain displacement relations are linear, except for ϵ for which the first and 2nd variations are:

2D: δϵ = δu’ + z’ δw’  
3D: δϵ = δu’ + y’ δv’ + z’ δw’

2D: δ1δ2ϵ = δ1w’ δ2w’  
3D: δ1δ2ϵ = δ1v’ δ2v’ + δ1w’ δ2w’

B-Matrix in δϵ = B δqc is same as B matrix for nelmt25 (see <nelmt25.docx> for 3D case)

## General Approach

δϵ = B δqc qc = Bc q

Last equation in indicial notation with implied summation of repeated index j:  
qci = Bcij qj where   
Bcij = δ( i, Icj) where δ(i,j)=1 if i=j, 0 otherwise; and

2D: [Ic1 ... Ic8 ]T = [ 1 2 3 1 4 5 6 4 ]T3D: [Ic1 ... Ic12 ]T = [ 1 2 3 4 5 1 6 7 8 9 10 6 ]T

## Kinematic Hardening Model for Shear Transfer

Plasitic slip defn: τ = G (s – spl)  
Yield Criterion |τ – τsh| ≤ τy , τsh’= Gpl spl’   
Consistency condition during yielding: τ’ = τsh’ = G (s’ – spl’)   
 ⇒ τ’ = GT s’ where GT =G [1 – G/(G+Gpl)] = G Gpl/(G+Gpl)

## end

# Implementation Notes

1. Material stiffness array D is renamed to DM. This includes contributions from axial stresses in the concrete only. It corresponds to the strain vectors:  
   2D case strains ϵ = [ϵ κ γ ]T  
   3D case strains ϵ = [ϵ κy κz γy γz ]T  
   However, there is no resistance against the shears γ. So the elements of DM corresponding to the shears are zero. I.e.,  
   2D: DM(i,3)=DM(3,i)=0 ∀ i  
   3D: DM(i,4)=DM(4,i)=0, DM(i,5)=DM(5,i)=0 ∀ i  
   This means that a smaller array DM would be sufficient, but nevertheless the shear components are included with zeroes in DM for similarity with nelmt25.
2. In nelmt25, CQZ is the shear in the steel. In nelmt22 it is the shear arising from inclination of the axial force N. Thus different meanings.

# Examples

## Pipe Under Inclined Load

A simply supported and axially restrained pipe is subject to transverse and axial distributed loads. The direction of the transverse load is varied to enable so that is can be checked that the response is isotropic and conincides with the 2D case.

The units are kips for force, inches for length, and consistent combinations thereof.

The model involves 4 joints of 24inch OD by 1inch WT pipe, coated with 2” of concrete, with 0.1” corrosion coating. The joint length if 40ft (480inch), and cut-back of the coating is 1ft (12inch)

# References

1. Ness, O.B and Verley, R., (1995), "Strain Concentrations in Pipelines with Concrete Coating: An Analytical Model," Proc. OMAE'95, Copenhagen, Denmark.
2. Verley, R., and Ness, O.B., (1995), “Strain Concentrations in Pipelines with Concrete Coating: Full Scale Bending Tests and Analytical Calculations,” Proc. OMAE 1995, Vol. 5, Pipeline Technology, ASME.
3. “Strain Concentrations in Pipes due to Concrete Coating,” by R. Peek, Shell International Exploration and Production, (SIEP EPT-PER), SIEP Report No. EP 2006-5161, April 2006. Released as unrestricted report, September 2016.