# Nelmt29: Beam with Constant Axial Force



Moderate deflection theory is used. The axial force N is taken as the 5th element degree of freedom. This degree of freedom should be conneced to a single node for all elements in the beam with constant axial force. The element is assumed to be inextensional (i.e. zero axial strains).

For inextensional virtual deformations the principle of virtual work can be written as

∫ (Q δγ + M δκ ) dx = ∫ fz δw dx - N δG + δWMQends (1)

where internal and external virtual work appears on the left and right sides, respectively; the integrals are taken over the entire beam over which the axial force N is constant; the shear deformation γ and curvature κ are given by

γ = w’ + θ , κ = θ’ (2a,b)

where w=w(x) and θ=θ(x) denote the lateral displacement (postive in the z direction), and rotation respectively (the rotations are positive in the direction defined by right-hand rule about the y-axis, where x,y,z are proper orthogonal directions); δWMQends is the external virtual work done by the external applied moment and shear at the ends, and

G = ∫ ε dx , ε = Z’w’ + ½ w’2 (3a,b)

denotes the total geometric shortening, where Z=Z(x) describes the initial (undeformed) geometry of the beam.

Augmenting Eq. 1 with Eq. 3a gives:

∫ (Q δγ + M δκ + N δε + δN ε ) dx = ∫ fz δw dx + δN G + δWMQends (4)

To evaluate the element contribution to the left hand side of this equation the following assumptions/approximations are made:

* The displacement w=w(x) and rotation θ=θ(x) varies linearly within each element.
* A single integration point is used at the centre of the element.

One can then write the element contribution to the left hand integrals

∫element (Q δγ + M δκ + N δε + δN ε ) dx = δqT s (5)

where

q = [ q1 q2 q3 … q5 ]T, s = [ s1 s2 s3 … s5 ]T (6)

q5 = N, s5 = g = ∫element ε dx = ∫element (Z’w’ + ½ w’2) dx (7a,b)

To derive this result and develop and expression for s, the shear strain and curvature at the integration point are first written as

γ = γqT q, κ = κqT q (8)

Taking variations then yields

δγ = γqT δq, δκ = κqT δq, δε = N z’ w’qT δq, δN = NqT δq (8a-d)

where

z’ = Z’ +w’ (9)

is the slope in the deformed configuration, and

γq = [ -1/L ½ 1/L ½ 0 ]T, κq = [ 0 -1/L 0 1/L 0 ]T (10a,b)

w’q = [ -1/L 0 1/L 0 0]T, NqT = [ 0 0 0 0 1]T (11a,b)

Thus,

∫element (Q δγ + M δκ + N δε + δN ε ) dx = δqT L (Q γq + M κq + N z’ w’q +ε Nq ) (12)

from which it follows that

s = L (Q γq + M κq + N z’ w’q +ε Nq ) (13)

The tangent stiffness is obtained by taking the derivative with respect to the components of q. It is assmued that Q depends on γ only and M depends on κ only (uncoupled section resistance), so that

k = L { γq Qγ γqT + κq Mκ κqT + N w’q w’qT + z’ (w’q NqT + Nq w’qT) } (14)

where

Qγ = dQ/dγ, Mκ = dM/dκ (15)

are the tangent stiffnesses for shear and bending deformations, respectively.

In summary:

1. Eqs. 13, and 14 given the element resistance vector s, and tangent stiffness matrix k, respectively. These are formed and assembled in the usual way for finite element calculations. It can be seen from Eq. 14 that the tangent stiffness matrix is symmetric.
2. The discretization is constructed in the usual way with 2 dof per node. However an additional axial force/deformation node should be added. For computational efficiency (skyline of tangent stiffness matrix) this should be the highest numbered node. Thus the skyline of the matrix only reaches the top of the matrix for the last degree of freedom, and is unaffected by the extra node for all other degrees of freedom.
3. Transverse loads and moments are applied in the usual way.
4. For the axial force/deformation node the following applies for the first degree of freedom (dof):
   1. The applied “load” becomes a specified amount of shortening for the beam.
   2. If the dof is fixed, the imposed “displacement” represents the applied axial load.
   3. By attaching a spring to the dof one can specify linear combinations of axial load and displacement.
5. All other dofs for the axial force/deformation node must be fixed. They are not used.

## Illustration of Cyclic Moment-Curvature Relation



See file npex.docm for description.

## Illustration of Plastic Hinging



To go from the reference moment-curvature diagram the indicated distance AB must be increased by a factor of L/LG.